OUGHTON PRIMARY & NURSERY SCHOOL

Written Calculation Policy

Policy Approval Date: September 2021 (originated 2014)

Approved by the Full Governing Body see minutes September 2021

Review Date: September 2022

This policy was written with guidance from HfL Maths Advisors with the Maths Co-ordinator in April 2014.

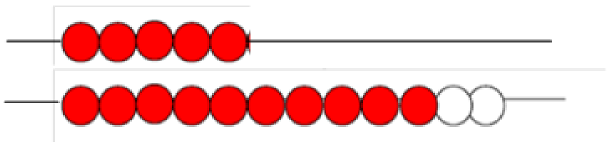
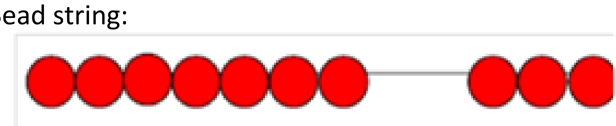
This policy outlines the progression through **written strategies** for addition, subtraction, multiplication and division. The aim is for children to become fluent in these written strategies through varied and frequent practice so that pupils develop conceptual understanding and the ability to recall and apply knowledge in a range of contexts. In addition to **fluency** of calculation teachers also need to consider frequent opportunities for children to **reason** using mathematical language as well as **solve problems** by applying their mathematics.

Children will move through the stages of written calculation at the pace appropriate to them however we expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum 2014. The policy includes examples and diagrams showing how to teach calculations as consistency in layout and presentation is important. The policy also includes the equipment and resources that will be used to support children's understanding of each strategy.

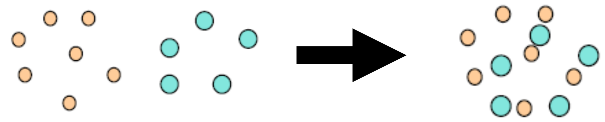
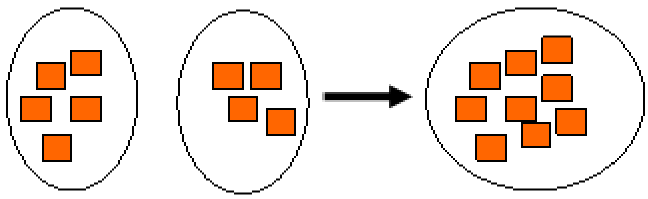
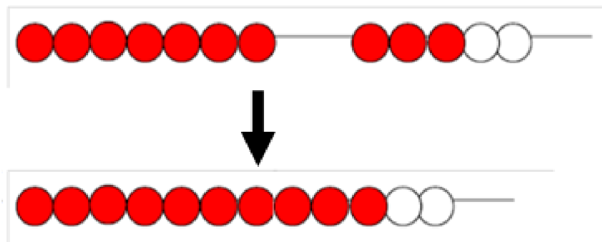
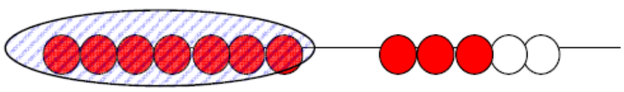
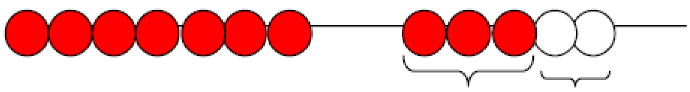
This policy focuses on written calculation in maths. It is also important to teach mental strategies alongside, which is done in accordance with the Herts for learning document Progression in Mental Mathematics and the Herts Essentials maths planning and fluency documents.

Counting and partitioning

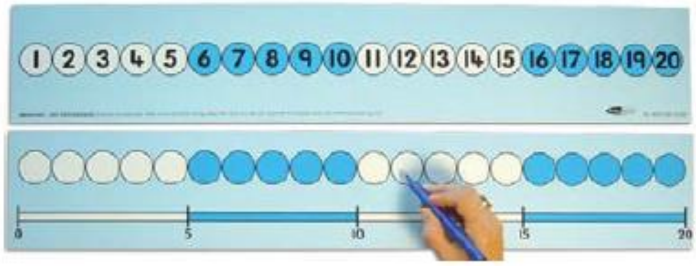
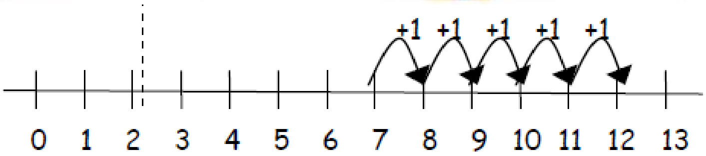
Counting: 1. One to one correspondence	1. Children synchronise their counting and pointing, keeping track of their counting as they go, assigning one number name to one object and only counting each object once. Counting static pictures is harder and children need to devise a system to know which they have counted as they go along.
2. Stable order of counting	2. To be able to count means knowing that the list of words used must be in a repeatable order. This principle calls for the use of a stable list that is at least as long as the number of items to be counted; if children only know the number names up to 'six', then they obviously are not able to count seven items.
3. Cardinal aspect of number	3. This is the idea that when they are counting a set of objects, the last number counted is the number of objects altogether.

4. Abstract principle of number	4. This is where children are counting things that cannot be touched or moved, such as sounds, imaginary objects or even the counting words.
Greater than/less than/equivalent	Using direct comparison with manipulatives – which is more? Bead strings:  Numicon:  Balance scales: 
Partitioning	1. Complements to 1, 10, 100 Bead string:  Numicon:  2. Partitioning any number in all possibilities e.g. partitioning 9: 0 + 9 1 + 8 2 + 7 3 + 6 4 + 5

Addition

Combining and counting (aggregation)	Count one set, then the other. Combine the sets and count again starting from 1. Using mixed sets of objects:  Count each set Combine and count Dienes units:  Count each set Combine and count Bead strings:  Count each set, combine and count
Combining and counting *on* (augmentation) **NB. IMPORTANT NOT TO FORGET THIS STEP**	Count one set, then count on from that set. Prepare for this stage with lots of play – count one set, hide it in bags, behind back, teddy eating them etc. Before counting on. Bead string:  Count 7, then count on 8, 9, 10, 11, 12
Bridging through 10	1. Manipulative – bead string:  7 + 5 How many more to the next multiple of 10? 3 If we use 3 of the 5 to get to 10 how many more do we need to add on?

<table>
<tr>
<td></td>
<td>
2. Visualising bead string in their head
3. Pictorial representations of bead string

4. Link to number line



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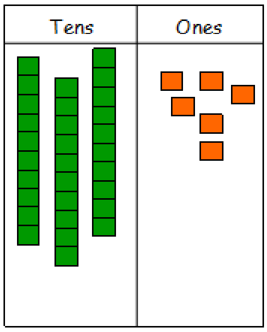
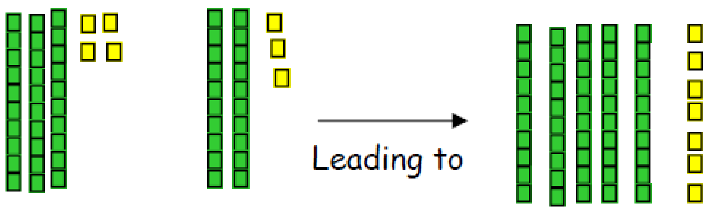
<tr>
<td>Transition to Dienes</td>
<td>
Familiarise the children with Dienes:
1. Compare 10s and 1s – lay units along the track of a ten. They need to grasp the relationship and equivalence.
2. Partition Tens and Ones using place value charts


3. Aggregation – combine the two sets, count <u>ONES FIRST</u> (starting from one), then tens (starting from ten)


4. Augmentation – as above but count from the first set of ones and tens, avoid starting at 1 i.e. start at 4 then continue 5, 6, 7.
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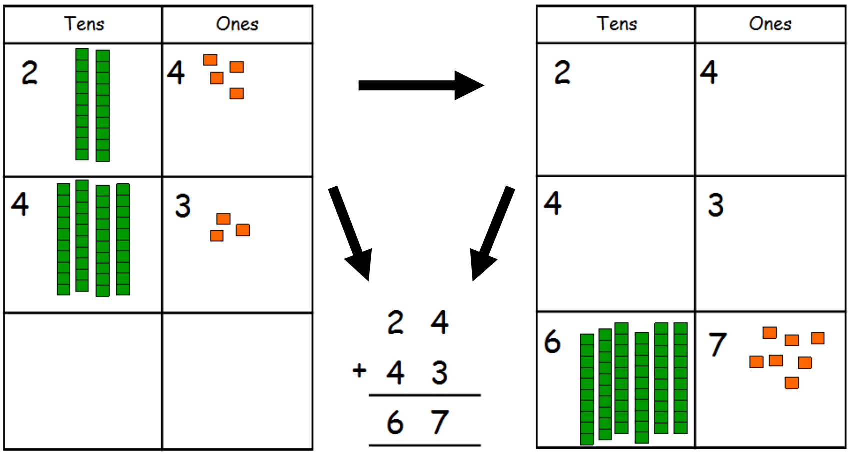
<table>
<tr>
<td>Columnar recording – no exchange</td>
<td>


Recorded on place value charts with Dienes, leading to without Dienes.
Combine **<u>Ones first</u>**, count and record, then Tens.

NB. For children who are struggling to understand this method, teachers may decide to introduce the expanded method
</td>
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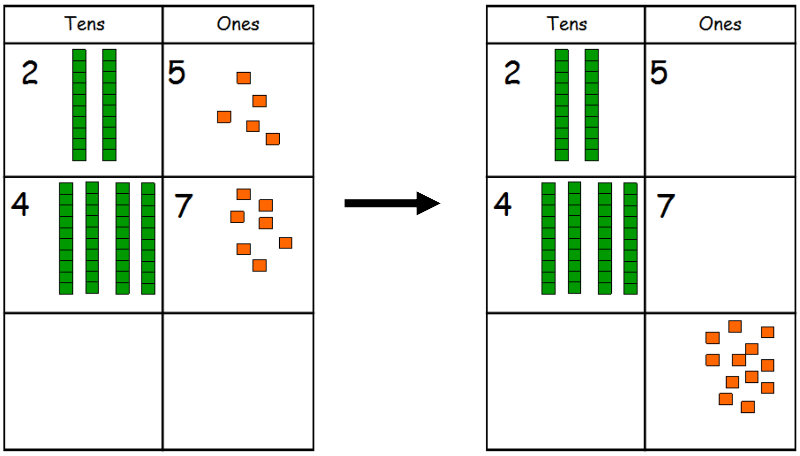
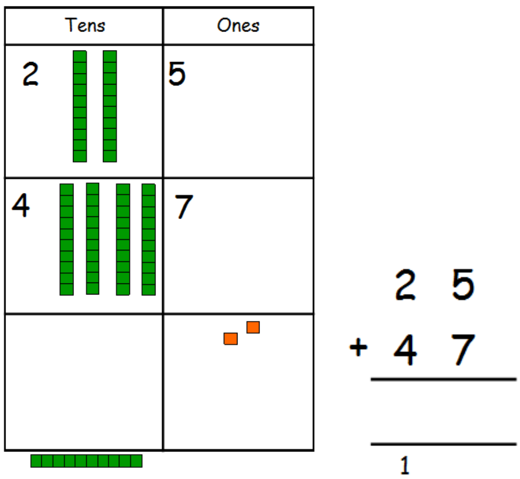
<tr>
<td>Columnar recording – exchanging</td>
<td>


Combine Ones



Exchange ten Ones for one Ten
Move the Ten to the next column – carrying underneath
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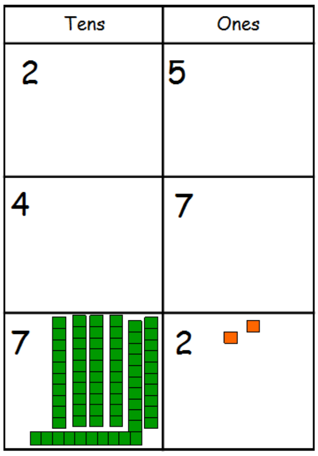
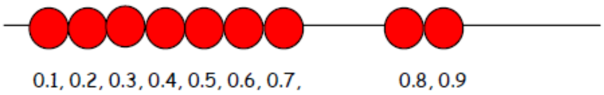
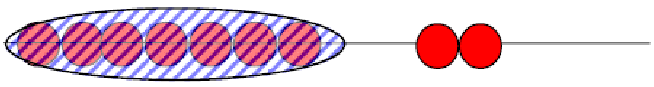
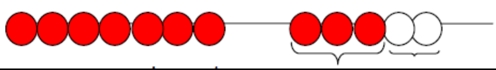
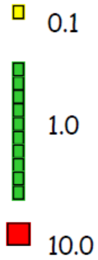
</tr>
</table>

	 $\begin{array}{r} 2\ 5 \\ +\ 4\ 7 \\ \hline 7\ 2 \\ \hline {}_{1}\ \ \ \end{array}$ Combine tens Run Dienes and place value chart method alongside compacted written method until children are confident.
Compacted written method	$\begin{array}{r} 2\ 5 \\ +\ 4\ 7 \\ \hline 7\ 2 \\ \hline {}_{1}\ \ \ \end{array}$ When children are confident without Dienes. Grading of difficulty: 1. No exchange 2. Extra digit in the answer 3. Exchanging Ones to Tens 4. Exchanging Tens to Hundreds 5. Exchanging Ones to Tens AND Tens to Hundreds 6. More than two numbers in calculation 7. Different numbers of digits 8. Decimals – see next stage
Decimals	It is important to take children back through the stages with decimals: 1. Aggregation:  Count both sets starting from zero 2. Augmentation:  Start from 0.7, count on 0.8, 0.9 3. Bridging through 10: 0.7 + 0.5 = 0.7 + 0.3 + 0.2 = 1.2 

	4. Columnar method with Dienes as decimals  5. Written columnar method as above.

Subtraction

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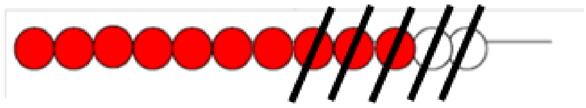
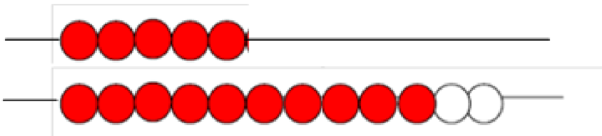
<tr>
<td>3 models of subtraction to be used throughout</td>
<td>

1. **<u>Take away/count back</u>**



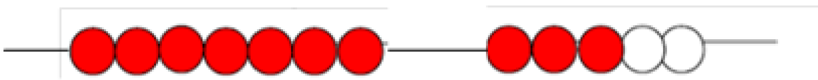
12 - 5
12 objects, count back 5

2. **<u>Comparing two sets/difference</u>**



12 – 5
Comparing 12 and 5, count difference

3. **<u>Partitioning</u>**



12 - 5
Seeing that 12 is made up of 7 and 5

</td>
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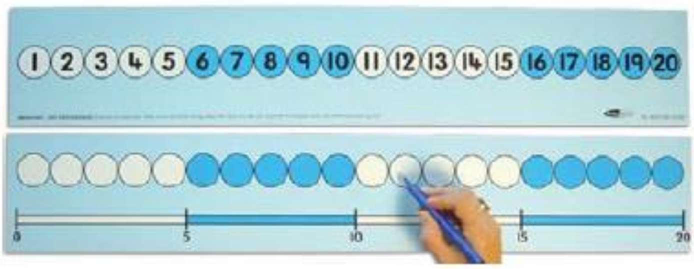
<tr>
<td>Subtracting single digits</td>
<td>

- Use manipulatives in the above 3 ways (Bead strings, numicon)
- Visualise bead string in their heads
- Pictorial representation of bead string

- Link to number line



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Familiarise the children with Dienes:

1. Compare 10s and 1s – lay units along the track of a ten. They need to grasp the relationship and equivalence.

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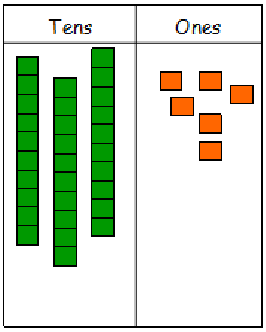
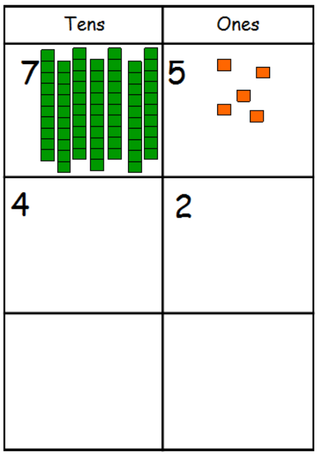
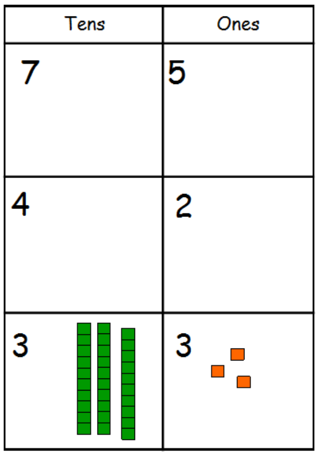
<table>
<tr>
<td></td>
<td>2. Partition Tens and Ones using place value charts
</td>
</tr>
<tr>
<td>TO – TO using Dienes and columnar place value charts – no exchange</td>
<td>

$$\begin{array}{r} 7\ \ 5 \\ -\ 4\ \ 2 \\ \hline \\ \hline \end{array}$$

Take the Ones first, then Tens


$$\begin{array}{r} 7\ \ 5 \\ -\ 4\ \ 2 \\ \hline 3\ \ 3 \\ \hline \end{array}$$

Record the answer underneath after moving the Dienes

NB. For children who are struggling to understand this method, teachers may decide to introduce the expanded method</td>
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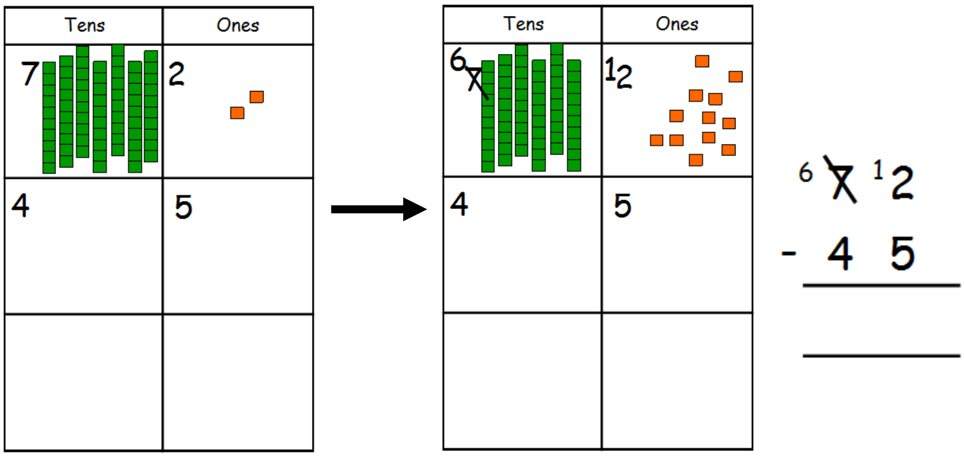
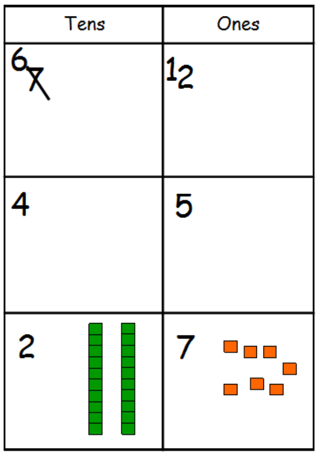
</table>

<table>
<tr>
<td>TO – TO using Dienes and columnar place value charts – with exchange</td>
<td>


Exchange one Ten for ten Ones, alter written numbers


$$\begin{array}{r} {}^{6}\cancel{7}\;{}^{1}2 \\ -\;4\;\;5 \\ \hline 2\;\;7 \\ \hline \end{array}$$

Take the <u>**Ones**</u>, *then* take the <u>**Tens**</u>

Once children are confident with the practical – record the compacted method *<u>alongside</u>* as shown.
</td>
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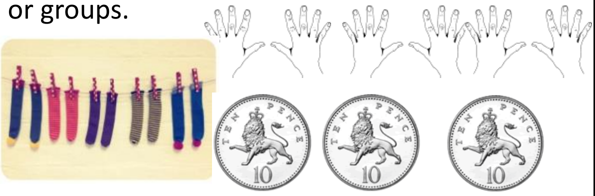
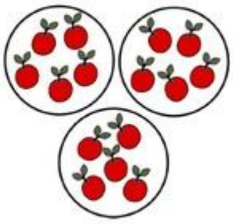
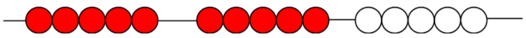
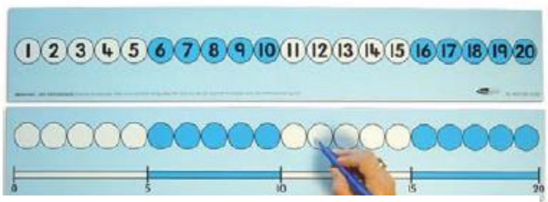
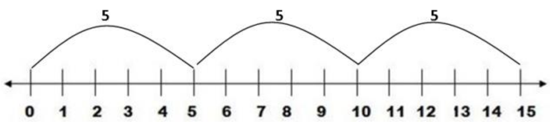
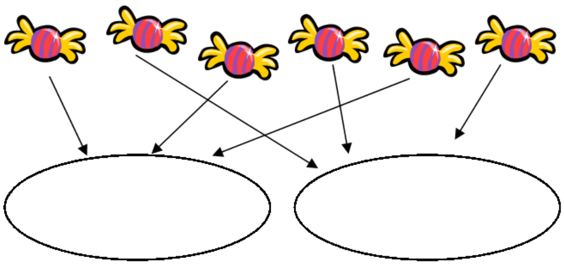
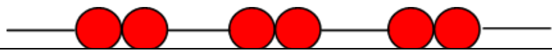
<tr>
<td>Column method</td>
<td>
When children are confident with Dienes and compacted method alongside, begin to record without Dienes.

$$\begin{array}{r} {}^{6}\cancel{7}\;{}^{1}2 \\ -\;4\;\;5 \\ \hline 2\;\;7 \\ \hline \end{array}$$

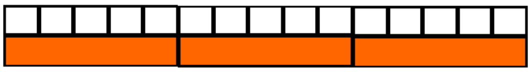
Grading of difficulty:

1. TO – TO, no exchange
2. TO – TO with exchange
3. HTO – TO with exchange
4. HTO – HTO with exchange
5. HTO – HTO with a zero in Tens column
6. Larger numbers
7. Decimals
</td>
</tr>
</table>

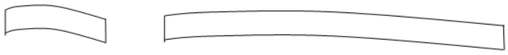
Multiplication and Division

Multiplication	**Division**
Early experiences Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups. 	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s. 
Repeated addition (repeated aggregation) 3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3 Children learn that repeated addition can be shown on bead string:  Number track and number line:  	**Sharing equally** 6 sweets get shared between 2 people. How many sweets do they each get?  **Grouping of repeated subtraction (NB important step – often forgotten)** There are 6 sweets. How many people can have 2 sweets each?  
Scaling This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you end up with 15 marbles. This can be written as:	**Repeated subtraction** Using a bead string, number track, then number line. 12 ÷ 3 = 4 How many 3s make 12? 

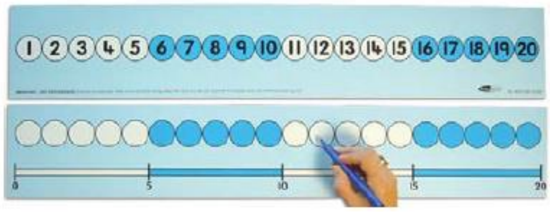
1 + 1 + 1 = 3 → Scaled up → 5 + 5 + 5 = 15

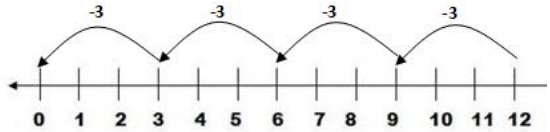


For example, find a ribbon that is 4 times as long as the blue ribbon.



5cm ribbon 20cm ribbon



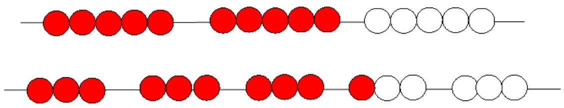


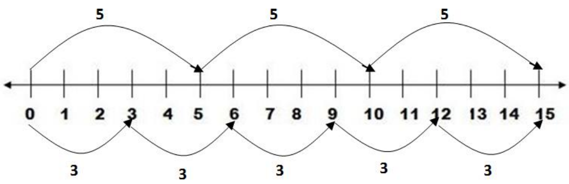
It is also important for children to have experience of different ways of this partitioning e.g. How many different ways can we make 12?

<u>Commutativity</u>

Children learn that 3 x 5 has the same total as 5 x 3. This can be shown on bead strings and number lines.

3 x 5 = 15
5 x 3 = 15



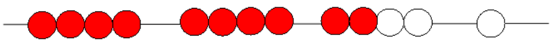


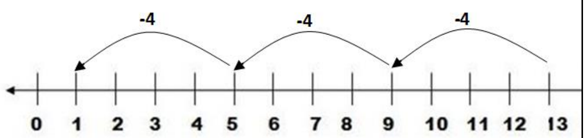
<u>Grouping involving remainders</u>

Children move onto calculations involving remainders.

If there are 13 sweets shared between 4 children, how many would each child get?

13 ÷ 4 = 3 r 1





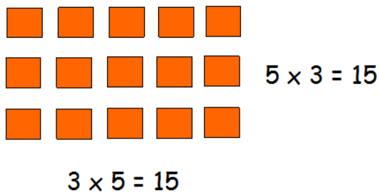
Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.
E.g. I have 62p. How many 8p sweets can I buy?
E.g. Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

<u>Arrays</u>

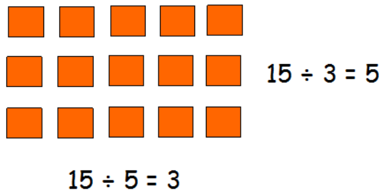
Children learn to model a multiplication calculation using an array. This model supports their understanding of commutativity and the development of the grid in a written method.

<u>Arrays</u>

Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the ‘bus stop method’ in a written method.



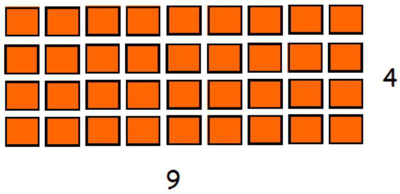
Children should be taught to arrange the array in different ways e.g. How many different ways can they be arranged?



Children should be taught to arrange the array in different ways e.g. How many different ways can they be arranged?

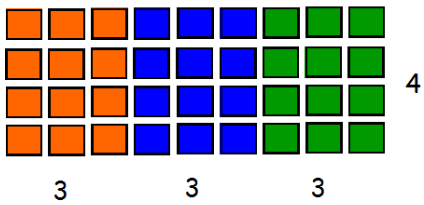
<u>Partitioning and the distributive law</u>

Arrays are useful to help children visualise how to partition larger numbers into more useful arrays.



9 x 4 = 36

Children could break this down into more manageable arrays (distributive law – New curriculum Y4):



9 x 4 = (3 x 4) + (3 x 4) + (3 x 4)
= 12 + 12 + 12
= 36

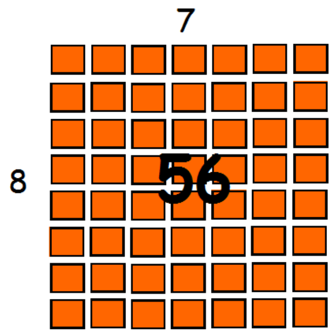
Children need to spend lots of time investigating different ways to partition using low numbers to embed this concept. (Children can use multilink cubes, Dienes blocks, squared paper, geo boards to make and split the arrays.)

e.g. How can 6 x 14 be partitioned? (investigate different ways)
One example:

6 x 14

<u>Partitioning and the distributive law</u>

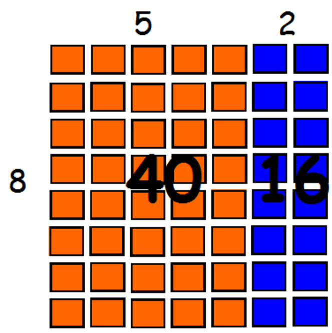
Arrays are useful to help children visualise how to partition larger numbers into more useful arrays.



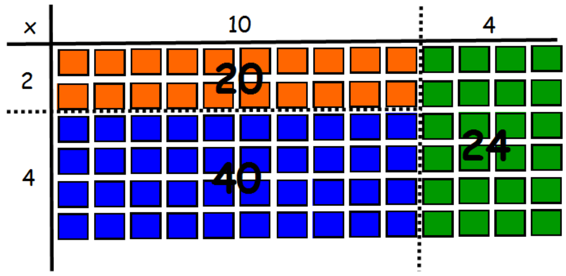
56 ÷ 8 = 7

Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between multiplication and division. (Distributive law – New curriculum Y4):

56 ÷ 8 =



56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8)
= 5 + 2
= 7



6 x 14

= (2 x 10) + (4 x 10) + (4 x 6)

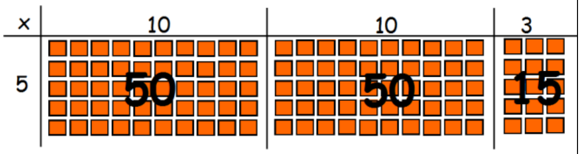
= 20 + 40 + 24

= 84

Children need to spend lots of time investigating different ways to partition using low numbers to embed this concept. (Children can use multilink cubes, Dienes blocks, squared paper, geo boards to make and split the arrays.)

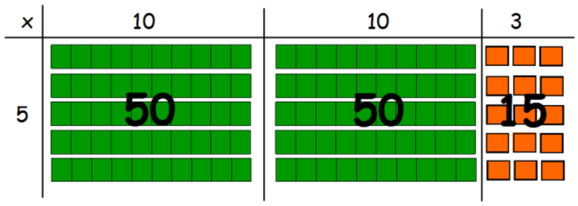
<u>Arrays leading to the grid method</u>

23 x 5



Transition to Dienes:

23 x 5



23 x 5 =

= (10 x 5) + (10 x 5) + (3 x 5)

= 50 + 50 + 15

= 115

Building to:

x	10	10	3
5	50	50	15

<u>Grid method</u>

Introduced for multiplication of TO x O to begin with.

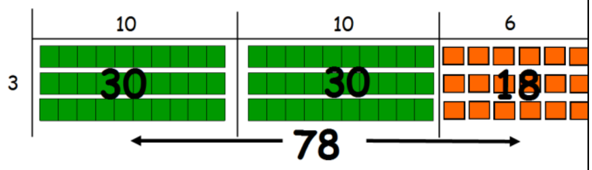
23 x 5 =

Children will approximate first.

<u>Arrays for division</u>

78 ÷ 3 =

(Children take 78 Dienes. Organise the tens into threes, then the ones, recording answer on the top)



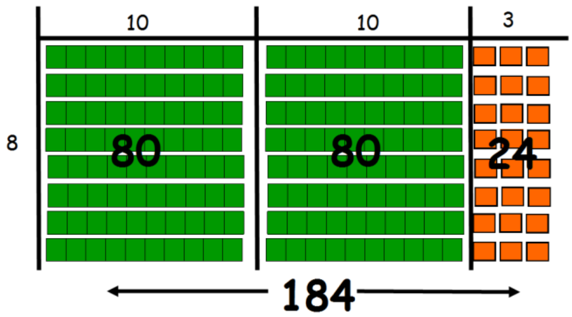
78 ÷ 3 =

= (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3)

= 10 + 10 + 6

= 26

184 ÷ 8



Building to:

184 ÷ 8

	20	3
8	160	24

x	20	3
5	100	15

100
+ 15
115

Moving to **<u>HTO x O</u>**

346 x 9

x	300	40	6
9	2700	360	54

2700
+ 360
+ 54
3114
1 1

<u>TO x TO</u>

72 x 38

x	70	2
30	2100	60
8	560	16

2100
+ 560
+ 60
+ 16
2736
1

<u>O.t x O</u>

4.9 x 3

x	4	0.9
3	12	2.7

12
+ 2.7
14.7

Children extend their use of the grid method to include:
ThHTO x O e.g. 4348 x 8
HTO x TO e.g. 372 x 24
O.th X O e.g. 4.92 x 3

Children are encouraged to record informal jottings alongside their work to help work out how to partition the 184 e.g.
1 x 8 = 8
2 x 8 = 16
3 x 8 = 24
20 x 8 = 160

<u>Short multiplication (To run alongside grid method)</u>
Children will refer back to the grid method and make the links between the methods. Links can also be made with place value boards and Dienes used in columnar addition.
24 x 6

<u>Long division 'The bus stop method'</u>
Building on arrays model above.

78 ÷ 3

	20	6
3	60	18

2 4 × 6 1 4 4 2	becomes... 2 6 3) 7 ^{1}8 432 ÷ 5 becomes 8 6 r 2 5) 4 3 ^{3}2
<u>Grading of difficulty (short multiplication)</u> 1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens 6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As above with greater digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As above with greater numbers of digits and decimals places.	**<u>Grading of difficulty (short division)</u>** 1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zeros in the quotient e.g. 816 ÷ 4 = 204 6. As 1 – 5 HTO ÷ O 7. As 1 – 5 greater number of digits ÷ O 8. As 1 – 5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3 **<u>Grading of difficulty for expressing remainders</u>** 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal
<u>Long multiplication</u> Children will refer back to the grid method and compare before recording as:	**<u>Long division</u>** 432 ÷ 15 becomes 2 8 r 12 1 5) 4 3 2 3 0 0 1 3 2 1 2 0 1 2

```
   1 0 7 4
x    2 2
   -------
         8
     1 4 0
     0 0 0
   2 0 0 0
       8 0
   1 4 0 0
   0 0 0 0
 2 0 0 0 0
 ---------
 2 3 6 2 8
```

```
   1 1
   1 0 7 4
x      2 2
 ---------
   2 1 4 8
 2 1 4 8 0
 ---------
 2 3 6 2 8
 ---------
     1
```

Children may then be taught that long multiplication can be done in a different order, depending on whether you start with ones or tens/hundreds. Children are encouraged to understand that both methods have the same processes.

24 × 16 becomes	124 × 26 becomes
 2 2 4 × 1 6 2 4 0 1 4 4 3 8 4 Answer: 384	1 2 1 2 4 × 2 6 2 4 8 0 7 4 4 3 2 2 4 1 1 Answer: 3224